Mathematics of Planet Earth

Classical and Modern Results in Nonlinear Filtering and Applications Workshop

Wednesday 28th November 2018, 09.30-17.00
Room 401, EPSRC Centres for Doctoral Training Suite at the South Kensington Campus, Imperial College London

This is a one-day workshop that aims to bring together researchers from two sister areas Data Assimilation and Nonlinear Filtering. The morning session will be devoted to will be on applications of filtering and methodology (in particular data assimilation). The afternoon theme will be on nonlinear filtering theoretical results.

Workshop Schedule:

9:30am	Coffee and registration
9:50am	Welcoming remarks
10:00am	David Mayne - *The life and work of Martin Clark*
10:20am	Wei Pan - *Sequential Monte Carlo for Stochastic Advection by Lie Transport GFD Models*
10:50pm	Coffee break
11:00am	Sebastian Reich - *Data Assimilation: Kalman's legacy and beyond*
11:45am	Etienne Memin - *Stochastic transport to track closed curves through image data*
12:30pm	Lunch
2:00 pm	Terry Lyons - *A new Lévy construction for Brownian motion and an application to the CIR model*
2:45pm	Mark Davis - *How can we tell whether probabilistic forecasts are correct?*
3:30pm	Coffee break
4:00pm	Richard Vinter - *Robust Filtering Algorithms for Tracking Problems with Measurement Process Nonlinearities*
4:45pm	Nigel Newton - *Nonlinear Filtering and Information Geometry*

Wei Pan

Title
Sequential Monte Carlo for Stochastic Advection by Lie Transport GFD Models

Abstract

We present an effective high dimensional data assimilation methodology for the damped and forced incompressible 2D Euler fluid flows driven by stochastic advection by Lie transport (SALT) type noise. SALT type stochastic partial differential equations (SPE) for fluids were introduced by [Holm, Proc Roy Soc, 2015]. According to [Holm, Proc Roy Soc, 2015] and [Cotter et al., 2017], the principles of transformation theory and multi-time homogenisation imply a physically meaningful, data-driven approach for decomposing the fluid transport velocity into its drift and stochastic parts, for a certain class of fluid flows. This results in a stochastic parameterisation for the fluid transport and gives us SPDE models for the fluid flows which preserve circulation.

A numerical methodology for implementing this velocity decomposition was developed in [Cotter et al., 2018] and applied to the 2D Euler equations, on a simply connected domain with no-penetration type bc, in consideration

$$d\omega + \left(\mathbf{u}dt + \sum_i \boldsymbol{\xi}_i \circ dW_t^i\right) \cdot \nabla\omega = f(\omega)$$

Successful uncertainty quantification results were obtained thus forming the ground work for this data assimilation study. Our SPDE model is prescribed on coarse resolutions, whereas the “original” deterministic partial differential equations are prescribed on fine resolutions. We consider two stochastic filtering data assimilation problems. In the first problem the observations come from a single realisation of the SPDE. In the second problem the observations correspond to the fine scale PDE system.

Sebastian Reich

Title
The unreasonable success of data assimilation

Abstract

The talk will provide an overview of data assimilation in the context of weather prediction and how the field has been shaped by the work of Kalman and modern computational advances such as ensemble prediction. Some recent extensions of the, so called, ensemble Kalman filter will also be discussed.

Etienne Memin

Title

Stochastic transport to track closed curves through image data

Abstract

We introduce a stochastic filtering technique for the tracking of closed curves from image sequence. For that purpose, we design a continuous-time dynamics that allows us to infer inter-frame deformations. The curve is defined by an implicit level-set representation and the stochastic dynamics is expressed on the level-set function. It takes the form of a stochastic partial differential equation with a Brownian motion of low dimension. The evolution model we propose combines local photometric information, deformations induced by the curve displacement and an uncertainty modeling of the dynamics. Specific choices of noise models and drift terms lead to an evolution law based on mean curvature as in classic level set methods, while other choices yield new evolution laws. The approach we propose is implemented through a particle filter, which includes color measurements characterizing the target and the background photometric probability densities respectively. The merit of this filter is demonstrated on various satellite image sequences depicting the evolution of complex geophysical flows.

Terry Lyons

(Joint with James Foster and Harald Oberhauser)

Title

A new Lévy construction for Brownian motion and an application to the CIR model.

Abstract

Levy gives a strong and recursive construction of space time Brownian motion. For the one-dimensional case there is second order parabolic equivalent that is easy to compute and strikingly effective in applications

Mark Davis

Title

How can we tell whether probabilistic forecasts are correct?

Abstract

This talk concerns checking the correctness of probabilistic forecasts. The main application is to risk measure estimates for financial data, for example value-at-risk (VaR) which forecasts a quantile of the return distribution. A single estimate followed by observation of the realized return does not give us much information but, using the statistical theory of elicitability, we can devise tests on a sequence of VaR-forecast/return pairs that will be passed if the forecasts are correct in some probabilistic model. Verifying forecasts of other risk measures such as Expected Shortfall is much more debatable.

Richard Vinter

Title

Robust Filtering Algorithms for Tracking Problems with Measurement Process Nonlinearities

Abstract

An important area of application of nonlinear filtering is Bayesian target tracking: estimating the state of a moving target from noisy sensor measurements. Here, linear discrete-time Gaussian models are commonly used to describe motion of the target and sensor platform. The estimation problem is typically nonlinear because of the nature of the measurement process which, depending on the precise nature of the application, provides noisy information about target angle-to-line-of-sight (or bearing), range or range rate, all of which quantities are nonlinear functions of target state.

We discuss a class of 'analytic' filters, i.e. tracking algorithms based on probability density calculations, not on the construction of empirical distributions (particle filtering). They are based on ideas of Martin Clark. Analytic filters cannot hope to match the versatility of particle filters. But they can provide excellent approximations for a significant class of tracking problems where the nonlinearities that arise occur in the measurement process equations alone. We describe a unified approach to the construction of filters for problems of this nature and examine in detail the specific form these filters take in numerous cases of interest: bearings-only tracking, range-only tracking and other instances where nonlinearities are present in the measurement equations.

Nigel J. Newton

Title

Nonlinear Filtering and Information Geometry

Abstract

The talk will briefly outline the connections between nonlinear filtering and information theory, before discussing a number of non-parametric information manifolds as candidate "state spaces" for a nonlinear filter. Such manifolds have appropriate topologies for approximations in multi-objective problems. They are also natural tools in the "stochastic dynamics" approach to nonequilibrium statistical mechanics, where information flows are associated with entropy production. Recent developments incorporate the topology of the signal space within that of the manifold.