Motivation

We can help tackle climate change by a more effective use of renewable energy sources. The goal of my research is to improve existing methods of stochastic modelling and statistical inference to quantify risk and uncertainty of renewable energy sources in a more reliable way.

Electricity features

Since electricity is generally traded for consumption, it is considered a commodity. However, in contrast to other types of commodities, it has some unique features ([1]).

• Non-storability (supply and demand must always match).
• Seasonality (higher demand in winter months due to the need of heating and longer use of lights).
• Periodic behaviour (higher demand in the peak time, i.e., Monday to Friday).
• Mean reversion (over time the electricity prices will tend to their average).
• Large and heteroscedastic volatility.

Empirical data

I work with a set of German data consisting of daily prices of spot and futures contracts over about 10 years.

The arithmetic model for spot prices

Let \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in \mathbb{R}}, \mathbb{P})\) be a probability space, where the filtration \(\{\mathcal{F}_t\}_{t \in \mathbb{R}}\) satisfies the ‘usual conditions’. Let \(S(t)\) be the spot price. Following [2], I proposed the arithmetic model

\[
S(t) = \Lambda(t) + Z(t) + Y(t),
\]

where \(\Lambda(t) + Z(t)\) is the long-term factor, while \(Y(t)\) describes the short-term behaviour, which includes the impact of renewables.

Model terms

- \(\Lambda(t)\) – a deterministic seasonality/trend function.
- \(Z(t)\) – a Lévy process with zero mean (under the physical measure).
- \(Y(t) = \int_{-\infty}^{t} g(t-s) \sigma_j \cdot dL_s\), with a kernel \(g(t-s)\) such that \(\lim_{t \to \infty} g(t-s) = 0\).

Here \(\sigma_1\) is a càdlàg stochastic process describing the volatility of \(Y(t)\). Similarly to [3], I defined it as

\[
\sigma_j = \int_{-\infty}^{t} j(t-s) dW_s,
\]

where \(j\) is a deterministic, positive function and \(V(t)\) – a Lévy subordinator. I assumed that \(\sigma_1\) is independent from the driving Lévy process \(L(t)\).

The arithmetic model for futures prices

By the no-arbitrage arguments, one can define the price of a futures contract with maturity \(T\) as

\[
f(t, T) = \mathbb{E}_Q[S(T)|\mathcal{F}_t],
\]

where \(0 \leq t \leq T < \infty\) and \(Q\) is a risk neutral probability measure (see eg. [2]).

In the arithmetic model case the forward price at the time \(t\) equals

\[
f(t, T) = \Lambda(T) + Z(t) + (T-t)\mathbb{E}_Q[Z(1)] + \int_{-\infty}^{T} g(T-s) \sigma_j \cdot dL_s
\]

\[+
\mathbb{E}_Q[L_1] \int_{t}^{T} g(T-s) \mathbb{E}_Q[\sigma_1|\mathcal{F}_s] ds.
\]

In the long run futures prices can be approximated by

\[
f(T) \approx \Lambda(T) + Z(T) + (T-t)\mathbb{E}_Q[Z(1)] + \mathbb{E}_Q[L_1] \left(\frac{\mathbb{E}_Q[\delta]}{\delta} \int_{0}^{\infty} g(y) dy\right).
\]

Current and future work

The most important step is to fit the proposed model to the empirical data in order to find the most appropriate form of \(\Lambda(t)\), \(Z(t)\) and \(Y(t)\). I am especially interested in the impact of renewables on electricity prices. Furthermore, I am going to compare the results to an alternative approach, in which one models futures prices directly using ambit processes (see eg. [4]).

References


