Dynamics of Oceanic Alternating Jets in the Presence of Zonally Varying Bottom Topography

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Motivation

Earth's oceans behave as a quasi-two-dimensional (2D) fluid on large-scales (tens of km or longer) as the vertical velocity component is of small magnitude in comparison to the horizontal velocity vectors. Because of this 2D nature, small turbulent eddies can have a global effect in the oceans. In fact, many large-scale circulations are formed because of the eddy action.

One such example would be the presence of alternating zonal jets in the oceans, which are hundreds of km long and once formed can remain in the oceans for months. Jupiter's stripes are formed by the same mechanism. These jets play an important role in ocean transport and mixing. Generally the jets are steady; however, they start to drift in the presence of varying bottom topography and the dynamics change significantly. These features aren't understood completely and studying jets-topography interactions will improve our understanding of the ocean circulation.

Introduction

- Alternating jets have been observed in the oceans in satellite altimetry data [Maximenko et al., 2005] and eddy permitting models.
- There is a general agreement that this phenomenon can be explained by simple Quasi-Geostrophic (QG) dynamics in the presence of meridional gradient of the Coriolis parameter [Rhines, 1975].
- Small-scale eddies interact and transfer energy to large scales (know as inverse cascade) to form these giant patterns.

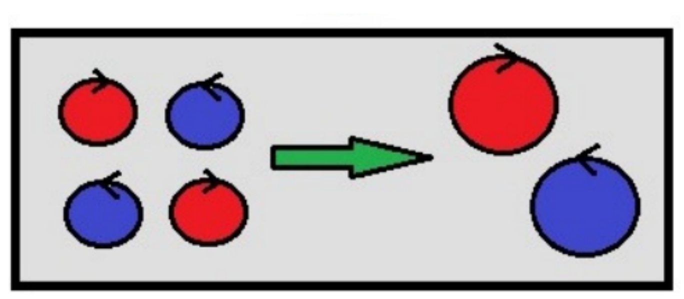

Figure 1: Evolution of vorticity field in the inverse cascade of energy.

- Many studies have given different mechanisms to explain the existence of alternating jets [Berloff et al., 2009a,b; Berloff, 2005; Dritschel and McIntyre, 2008].
- The jets tilt with increasing bottom slope in the zonal direction and also drift in the meridional direction [Boland et al., 2012].
- Transient jets have also been observed in the presence of wavy bottoms [Chen et al., 2015; Thompson, 2010].

Model Details and Results

A two-layer QG model is used where the top layer is forced with an eastward background flow. Bottom topography increases linearly in the zonal direction. Periodic boundary conditions are imposed on the boundaries in both directions. Spatial resolutions of 512x256 and 1024x512 are used in the simulations. Other parameters are same as in Berloff (2005).

Below are the governing equations,

$$\frac{\partial Q_1}{\partial t} + J(\psi_1, Q_1) + \beta \frac{\partial \psi_1}{\partial x} = \nu \nabla^4 \psi_1,$$

$$\frac{\partial Q_2}{\partial t} + J(\psi_2, Q_2) + (\beta + \frac{f_o}{H_2}\frac{\partial \eta_b}{\partial y})\frac{\partial \psi_2}{\partial x} - \frac{f_o}{H_2}\frac{\partial \eta_b}{\partial x}\frac{\partial \psi_2}{\partial y} = \nu \nabla^4 \psi_2 - \gamma \nabla^2 \psi_2.$$

$$Q_1 = \nabla^2 \psi_1 + S_1(\psi_2 - \psi_1) \quad Q_2 = \nabla^2 \psi_2 + S_2(\psi_1 - \psi_2)$$

From Figure 1, the alternating jets tilt and drift meridionally in the presence of sloped topography (consistent with Boland et al, (2012)). The steady state energy of the system also increases with increasing the bottom slope.

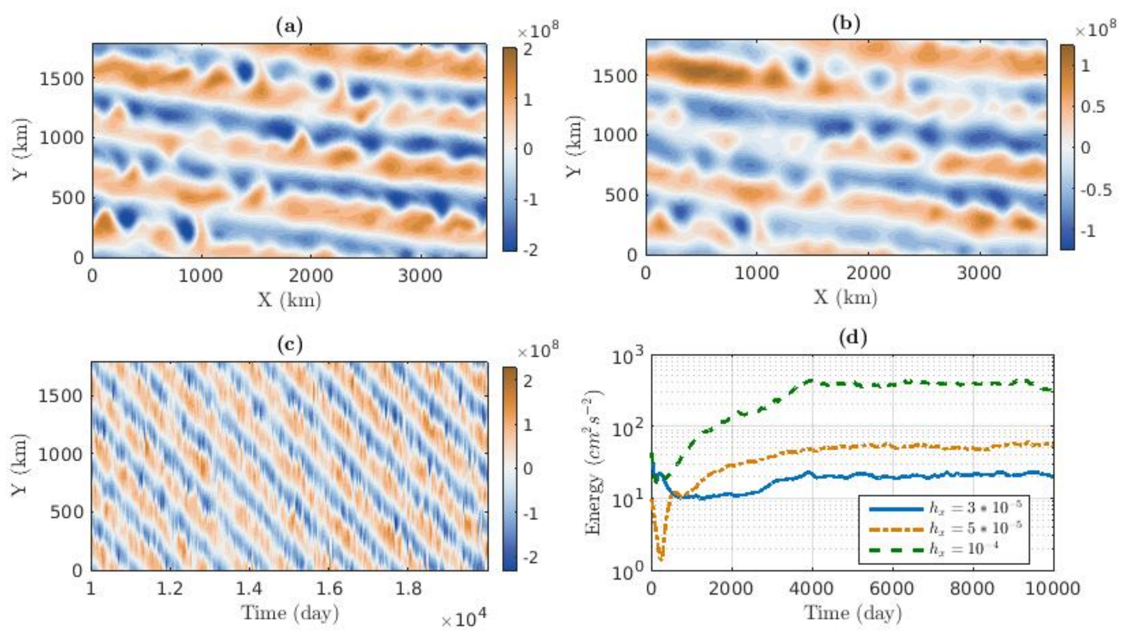

Figure 1: (a), (b) - Snapshots of streamfunction contours in the top and bottom layers (T = 9000 days); (c) - Hovmoller diagram of streamfunctio in the top layer, (d) - Total energy (spatial mean) time series for different values of zonal slope. All runs were performed on 512x256 grid.

Principle Component Analysis

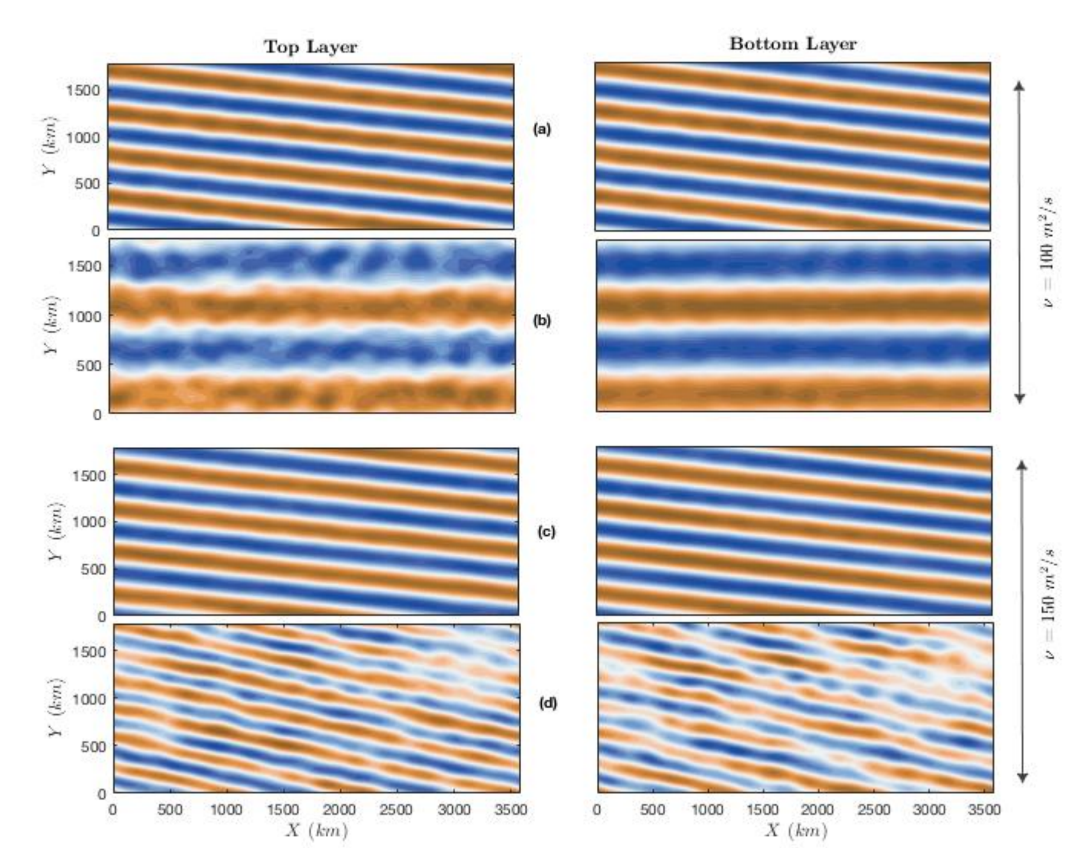

Figure 2: EOFs of streamfunction (data used between 8000 and 20000 days for the EOF analysis). ν = 100 m^2/s in (a) EOF1, (b) EOF3, and ν = 150 m^2/s in (c) EOF1, (d) EOF3. Colorbar range is [-1,1], blue to brown.

Streamfunction field is decomposed into a set of Empirical Orthogonal Functions (EOFs). As expected, first two EOFs capture the tilted jets. Interestingly, the jets become unstable and a purely zonal mode appears (EOF 3-4) when the eddy viscosity is 100 m^2/s; however, this doesn't happen in the case of higher eddy viscosity.

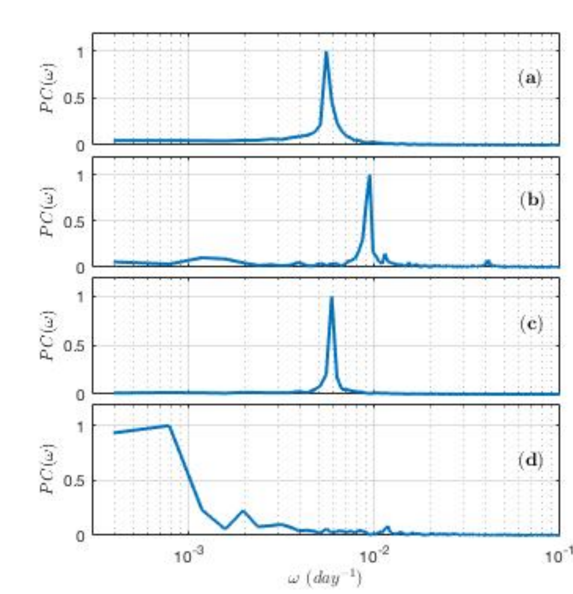

Figure 3: Spectra of PCs of streamfunction. ν = 100 m^2/s in (a) EOF1, (b) EOF3, and ν = 150 m^2/s in (c) EOF1, (d) EOF3.

$$c_i = \frac{\omega_i}{|\mathbf{k_i}|^2}\mathbf{k_i}$$

ν	Var, J (%)	Var, M (%)	Drift, J (cm/s)	Drift, M (cm/s)
200	81.4	0	-0.49	-
150	81.59	0	-0.49	-
100	67.76	7.51	-0.45	1.56
75	49.22	12.5	-0.42	1.50
50	26.94	12.37	-0.38	8.45

Table 1: Variances, drift velocities of the jets (J) and the zonal (M) modes for different values of eddy viscosity (similar behavior is observed if the bottom friction is decreased).

Linear Stability Analysis

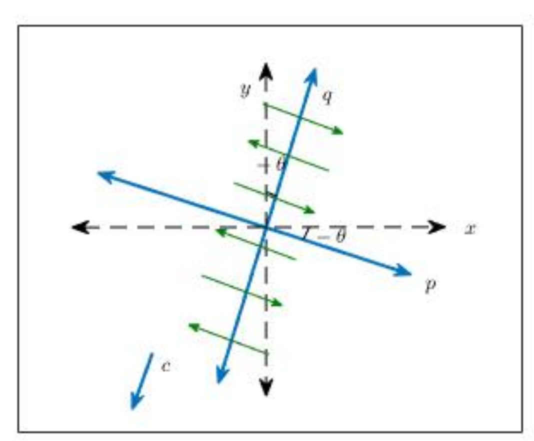

Figure 4: New frame of reference (new axes are p, q), rotated by an angle $\boldsymbol{\theta}$ and moving with speed c in q direction. Green arrows represent jets.

In order to better understand the effects of eddy viscosity and bottom friction, linear stability analysis is performed by linearizing the governing equations around the tilted jets. For that purpose, a drifting rotated frame of reference is used in which the jets are purely zonal and steady. Linearized equations can then be written as,

$$\frac{\partial}{\partial t}\left[\nabla^2\psi_1' + S_1(\psi_2' - \psi_1')\right] + \left[A_1\frac{\partial}{\partial p} + B_1\frac{\partial}{\partial q}\right]\left[\nabla^2\psi_1' + S_1(\psi_2' - \psi_1')\right] + C_1\frac{\partial \psi_1'}{\partial p} + D_1\frac{\partial \psi_1'}{\partial q} = \nu\nabla^4\psi_1'$$

$$\frac{\partial}{\partial t}\left[\nabla^2\psi_2' + S_2(\psi_1' - \psi_2')\right] + \left[A_2\frac{\partial}{\partial p} + B_2\frac{\partial}{\partial q}\right]\left[\nabla^2\psi_2' + S_2(\psi_1' - \psi_2')\right] + C_2\frac{\partial \psi_2'}{\partial p} + D_2\frac{\partial \psi_2'}{\partial q} = \nu\nabla^4\psi_2' - \gamma\nabla^2\psi_2'$$

where $A_1 = U_b\cos\theta + \overline{u}_1, B_1 = -c - U_b\sin\theta, C_1 = (\beta + S_1U_b)\cos\theta - \overline{u}_1'' + S_1(\overline{u}_1 - \overline{u}_2), D_1 = -(\beta + S_1U_b)\sin\theta$
$A_2 = \overline{u}_2, B_2 = -c, C_2 = (\beta - S_2U_b)\cos\theta - T_x\sin\theta - \overline{u}_2'' + S_2(\overline{u}_2 - \overline{u}_1), D_2 = -(\beta - S_2U_b)\sin\theta - T_x\cos\theta$

Linear Stability Analysis

The Mean velocity profile is constructed by averaging the solution field for 500 snapshots in the moving frame. Fourier solutions $\psi_i' = \tilde{\psi}_i(q)e^{i(lp-\omega t)}$ and finite difference discretization are used in p, q directions respectively. This results in an eigenvalue problem that is solved at all wavenumbers for different cases of ν and $\boldsymbol{\gamma}$.

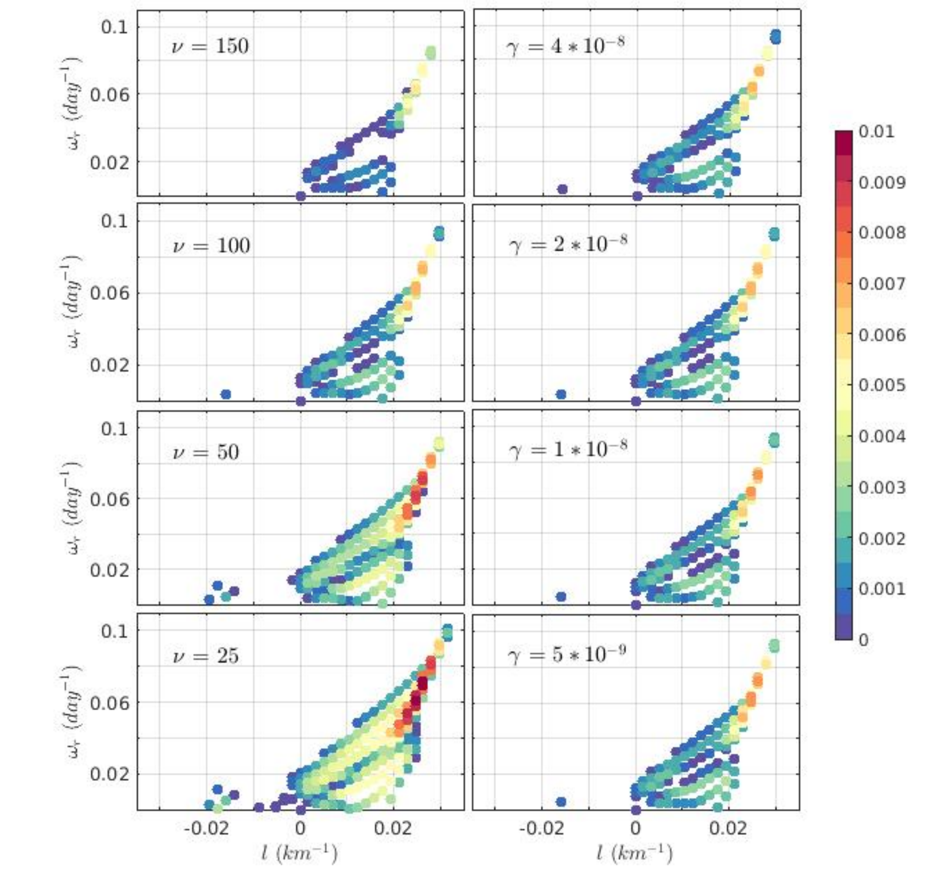

Figure 5: Real parts of eigenvalues for different values of eddy viscosity ($\boldsymbol{\gamma}$ =2*10^{-8} s^{-1} fixed) and bottom friction (ν = 100 m^2/s fixed) vs wavenumber (l=2n$\boldsymbol{\pi}$/L). Growth rates (day^{-1}) are shown in the colorbar.

Eddy viscosity and bottom friction tend to stabilise the system at all wavenumbers because the growth rates decrease with increasing ν and $\boldsymbol{\gamma}$. Large-scale modes are weakly growing, so the energy must be transferred to the M mode via nonlinear interactions.

Conclusions

- The jets can become unstable given that the viscous dissipation and friction terms are small and transfer energy further upscale to a purely zonal mode that propagates opposite to the jets.
- From linear stability analysis, it is clear that large values of viscosity inhibit this upscale transfer (from alternating jets to the zonal mode) by suppressing small-scale instabilities.
- This has also been verified by examining the kinetic energy spectrum.

Future Directions

- Investigation of the drift mechanism.
- Energetics of the QG system to understand the energy transfer between the jets and the zonal mode.

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