The model which we aim to study consists of the description of the motion of a fluid within a rotating frame $(\Omega, \Omega, \Omega)$ with angular velocity $\Omega_0 \kappa$ which occupies a domain delimited below by a fixed bottom $b(x,y)$ and above by a free surface, where the fluid is under the influence of the force of gravity $-g \kappa$. A typical realistic situation which would fit this model is for instance the zone of shallow sea water or the water in a fluvial canal. It has been noticed in literature (ADD REF) that in order to characterize completely the problem it is possible restrict the analysis to the wave elongation $h(x,y,t)$ form the characteristic height $h_0$ and the vertical averaged fluid velocity $\overline{u} = (\overline{u}_x(x,y,t), \overline{u}_y(x,y,t))$.

In order to emphasize the stationary state of the problem, the boundary $b(x,y)$ is supposed to be null.

The first equation is the mass continuity equation:

$$ \partial_t h + \mathbf{V} \cdot ((h + ho) \mathbf{U}) = 0. $$

After making use of the advective derivative $d_j(\mathbf{V}) = \partial_j((\mathbf{U} \cdot \mathbf{V}) \mathbf{V})$ and using Newton’s second law we get

$$ d_j \mathbf{U} + g \mathbf{V} h + \Omega_0 \kappa \times \mathbf{U} = 0 $$

In equation (2) we assumed the vertical component of the Coriolis force null and assumed the hydrostatic balance for the free surface, so that the pressure is a resulting function of the height $h$:

$$ p = p(h(x,y) - z) $$

where $p$ is the constant density of the fluid and so the term $-g \mathbf{V} h - \partial_z p$ is nothing but the gradient of the pressure. Equations (1) and (2) completely determine the dynamic of the fluid.

Deriving the operator associated to the linearised system

In the following we introduce the linearised system for the simplified system (1),(2) where the bottom function is supposed to be null $b(x,y) = 0$. The linearisation will be made around the trivial equilibrium where the elongation and the vertical averaged fluid velocity $\overline{U}$ are null.

The last system of equations (3) can be reformulated using a shorter notation as in the follow:

$$ d_j \left( \frac{\mathbf{U}}{h} \right) = - \left( \partial_j \Omega_0 \kappa \mathbf{v} + \frac{1}{2} \mathbf{V} \cdot \mathbf{V} \right) \mathbf{U} = \mathcal{S} \left( \frac{\mathbf{U}}{h} \right) $$

where the linear differential operator $\mathcal{S}$, defined on $\mathcal{H} = L^2(\mathbb{R}^2; \mathbb{R}^2 \times \mathbb{R})$ the Hilbert space of the complex square integrable vector-valued functions defined on $\mathbb{R}^2$, turns out to be antisymmetric, namely it verifies the following property:

$$ \mathcal{S}(a,b) = - \mathcal{S}(b,a) $$

The background image is a detail of the following picture.

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**Introduction to the Rotating Shallow Water Equations**

**Water waves and art**

**References**


