Potential vorticity redistribution in the forced shallow water system

Michael Haigh,
Department of Mathematics, Imperial College London

EPSRC Centre for Doctoral Training
Mathematics of Planet Earth
Imperial College London
University of Reading

Motivation and statement of the problem

Mesoscale eddies play a crucial role in driving the ocean circulation, but their turbulent nature means that general circulation models (GCMs) require horizontal grid resolutions on the order of 1 km to dynamically resolve them. Such fine grid resolutions may at times be feasible for short-time integrations, but are unrealistic for long-time integrations, such as those used in climate-related studies which require integrations of coupled atmosphere-ocean models over centuries and millenia.

To get around this issue, GCMs commonly employ a **parameterisation** which aims to accurately account for turbulent eddies and their effects on the large-scale flow. Parameterisations in GCMs often either:

1. use a **stochastic forcing** term to represent eddies (Porta Mana and Zanna, 2014),
2. make the assumption that turbulent eddies essentially act as a **diffuser of tracers**, a formulation pioneered by Gent and McWilliams (1990),
3. or use a combination of the two methods (Jansen and Held, 2014).

Here we introduce the early stages of an alternative method (similar to Berloff (2015)), which involves solving the shallow water equations and analysing how elementary eddy-like forces act to distribute potential vorticity (PV) in the system.

The shallow water system

The equations

The 1-layer shallow water equations, with viscosity and bottom friction, linearised about a purely zonal background velocity $U_0(y)$ and corresponding geostrophic sea-surface height (SSH) $H_0(y)$, are

$$Ro\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial x}\right)\mathbf{u}'+\mathbf{f}\times\mathbf{u}'+\hat{\mathbf{j}}Ro\frac{\mathrm{d}U_0}{\mathrm{d}y}v'=-\nabla\eta'+\frac{Ro}{Re}\nabla^2\mathbf{u}'-\gamma\mathbf{u}'+(F_1,F_2),$$

$$\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial x}\right)\eta'+\frac{\mathrm{d}H_0}{\mathrm{d}y}v'+H_0\left(\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right)=F_3,$$

where $\mathbf{u}'=(u',v')$ is the horizontal velocity anomaly and η' is the SSH anomaly. $Ro\approx 0.002$ is the Rossby number, $Re\approx 4\times 10^4$ is the Reynolds number and $\mathbf{f}=f(y)\hat{\mathbf{k}}$ is the linear Coriolis parameter. We assume that the dynamics is **periodic in time**, so $F_3(x,y,t)\to F_3(x,y)\exp(i\omega t)$, for example, for a forcing/solution frequency ω.

The forcing

We choose the forcing F_3 to be a cosine-shaped disturbance to the SSH extending to some radius r_0, centered at $\mathbf{x}_0=(x_0,y_0)$. Also, define $F_{1,2}$ so that the forcing regime itself is in geostrophic balance. Then we have:

$$F_3(x,y)=\begin{cases}A\cos\left(\frac{\pi}{2}\frac{r}{r_0}\right) & r\le r_0,\\ -\varepsilon & r>r_0,\end{cases}$$

$$F_1=-\frac{1}{f}\frac{\partial F_3}{\partial y},\quad\text{and}\quad F_2=\frac{1}{f}\frac{\partial F_3}{\partial x},$$

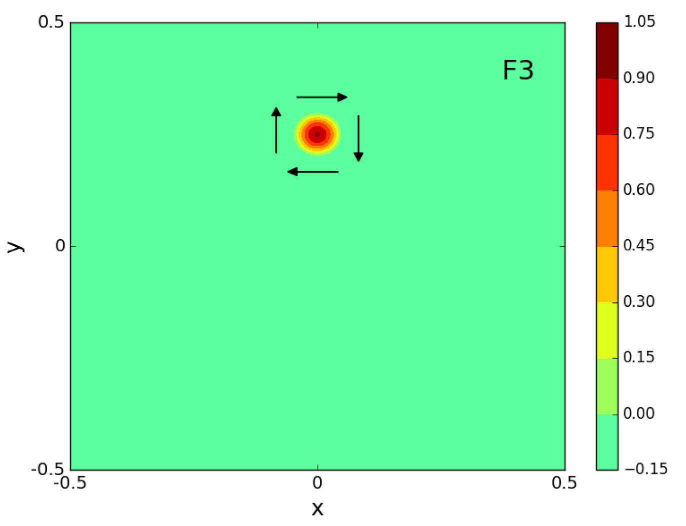

Figure: A snapshot of the forcing on the continuity equation centered at $\mathbf{x}_0=(0,0.25)$. Arrows show the direction of rotation. For presentation purposes, $r_0=180/3840$ in this plot, twice the value used in the solutions.

where $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$. The parameter $\varepsilon\ll A$ ensures conservation of mass at all times for some arbitrary forcing amplitude A.

Typical solutions

The system is solved in a zonally periodic channel, with no-normal-flow and free-slip conditions at the northern and southern boundaries.

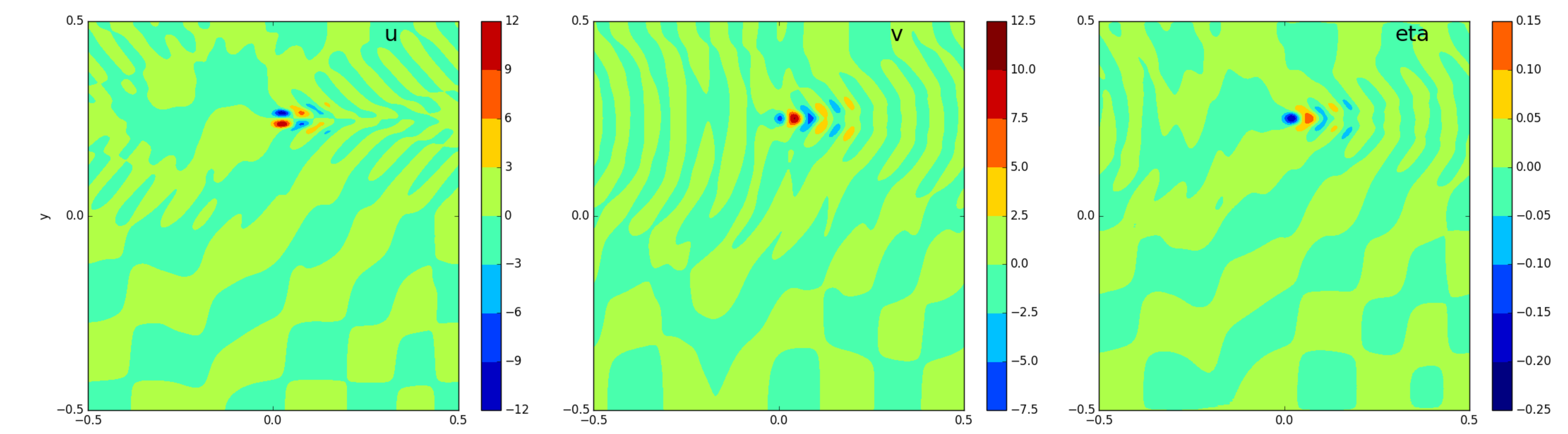

Figure: 1-layer shallow water solution snapshots for a uniform, eastward background flow $U_0=0.1$ (m s^{-1}). Plotted is the (normalised) zonal velocity (left), meridional velocity (center) and SSH (right) anomalies. The forcing is located in the northern half of the domain with 60-day period.

Animations of the solutions show that the disturbance introduced by the forcing is advected with the background flow, and, in the far-field, there is westward Rossby wave propagation with group speed ~ 0.1 m s^{-1}.

PV redistribution - footprints

We are interested in how the external forcing redistributes PV in the system. To this end, we define the **time-mean PV flux convergence**, or *footprint*, to be:

$$P\equiv-\nabla\cdot\overline{q\mathbf{u}},\quad\text{where}\quad q=\frac{\zeta+f}{\eta}$$

defines the potential vorticity, $\zeta=\partial_x v-\partial_y u$ is the relative vorticity, and the overbar denotes a time average. The figure below plots a typical footprint (corresponding to the previous solutions) and its zonal average $\langle P\rangle$. A typical footprint consists of a strong positive/negative 'pool' to the north/south of the forcing location, a quality corroborated by the zonal average.

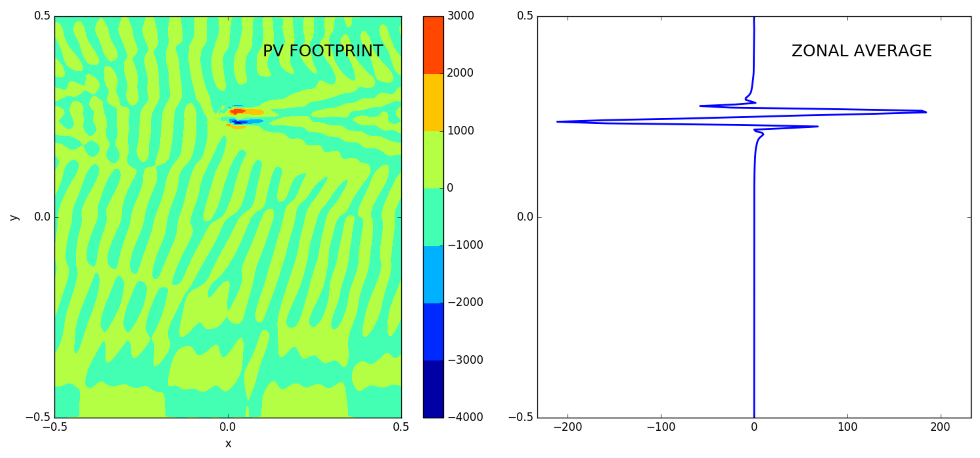

Figure: PV footprint (left) and the zonal average (right) for the reference solution.

A decomposition of the footprint into its components indicates that the dominant contribution to the zonal average comes from qv, the meridional PV flux. Although qu is of a similar magnitude, average values are two orders of magnitude smaller than that of qv.

Equivalent eddy fluxes

We are most interested in the meridional redistribution of PV, so define the equivalent eddy flux (EEF) as follows

$$\mathcal{P}\equiv\mathcal{P}_N+\mathcal{P}_S,\quad\text{where}\quad\mathcal{P}_N\equiv\omega\int_{y>y_0}\langle P\rangle\,dy\cdot\frac{\int_{y>y_0}|y|\,|\langle P\rangle|\,dy}{\int_{y>y_0}|\langle P\rangle|\,dy}.$$

$\mathcal{P}_S$ is defined in the same way as $\mathcal{P}_N$, but with $y<y_0$ in the integration limits. The idea is that $\mathcal{P}$ quantifies the extent of the meridional PV redistribution. Its components $\mathcal{P}_S$ and $\mathcal{P}_N$ are the meridionally integrated footprint zonal average (taken either side of the forcing), multiplied by a 'center of mass' and the forcing frequency for normalisation.

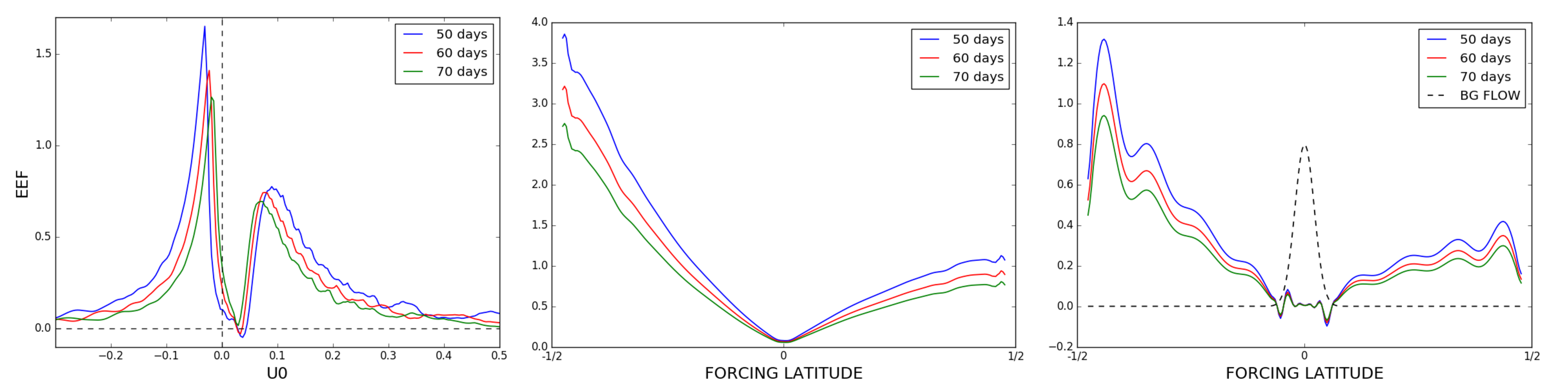

Figure: Plots of the equivalent eddy flux. (1) Left: EEF against uniform BG flow speed U_0 and northern forcing location. (2) Center: EEF against forcing latitude for a uniform BG flow of $U_0=0.1$. (3) Right: EEF against forcing latitude with a Gaussian jet BG flow with maximum speed 0.4, represented by the dotted line. In each panel, the EEF is plotted for three forcing periods, indicated in the legends.

(1) The EEF is positive for almost all uniform BG flows, with negative values near $U_0=0.03$ for 60- and 70-day forcing periods. There are two maxima: one for positive U_0 and one for negative. (2) For a uniform BG flow, a centrally located forcing produces the weakest PV redistribution. Moving the forcing northward produces a stronger EEF, and moving it southward induces an EEF approximately three times stronger than this. (3) For a Gaussian BG flow we observe similar behaviour for northern and southern $\mathbf{x}_0$ as in the uniform BG flow case, but here instead the y-inhomogeneity of U_0 introduces oscillations in the EEF curves that grow larger as $\mathbf{x}_0$ approaches a boundary. The EEF also has negative values at the 'edges' of the jet.

Future steps with equivalent eddy fluxes

The EEF and its dependence on the background flow will provide the basis for a novel parameterisation. The idea is to use the EEF to scale a series of PV dipoles that are external sources/sinks of PV in a non-eddy-resolving GCM.

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mh115@ic.ac.uk