Analyzing the UK energy and European carbon markets using forward-backward stochastic differential equations (FBSDEs)

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Background

- Carbon markets are currently being implemented world-wide to mitigate the effects of climate change. The EU Emissions Trading System (EU ETS) is the most prolific example.
- We consider a model for an energy market that produces electricity and is connected to a carbon market. For each ton of CO2 emitted, energy producers must submit an allowance at the end of the period; for any additional emissions, firms must pay a high penalty (€100 in the EU ETS).
- The processes, functions and parameters used will represent the UK energy market. Throughout, we use real data to estimate parameters.

Introduction

At time $\mathbf{t}$, set: $\mathbf{W_t} = (\mathbf{W^D_t}, \mathbf{W^C_t}, \mathbf{W^G_t})$: 3D standard Brownian motion process, $\mathbf{S^D_t}$ - demand for electricity, $\mathbf{S^C_t}$ - coal price, $\mathbf{S^G_t}$ - gas price, $\mathbf{E_t}$ - cumulative emissions, $\mathbf{A_t}$ - emissions allowance price.
The model for the market is represented by a FBSDE. Let

$$\mathbf{S^i_t} = \exp(\mathbf{h^i}(\mathbf{t}) + \mathbf{X^i_t}),$$

where $\mathbf{h^i}$ represents seasonal factors in the market, and $\mathbf{X^i}$ is a mean-reverting process with mean zero that is adapted to $\mathbf{W^i_t}$. The pricing FBSDE is, for $\mathbf{t} \in [\mathbf{0}, \mathbf{T}]$:

$$\begin{aligned} d\mathbf{E_t} &= \mu_E(\mathbf{S^D_t}, \mathbf{S^C_t}, \mathbf{S^G_t}, \mathbf{A_t})dt \\ d\mathbf{A_t} &= \mathbf{Z_t} \cdot d\mathbf{W_t}, \end{aligned} \qquad (1)$$

where

$$\mathbf{E_0} = \mathbf{0}, \quad \mathbf{A_T} = \mathbf{\Lambda 1}_{[\mathbf{E^{cap}},\infty)}(\mathbf{E_T}).$$

Here, $\mathbf{\Lambda}$ is the penalty, and μ_E the market emissions rate. We set $\mathbf{\Lambda} =$ £83.47. (equal to €100 in January 2012). The cap, $\mathbf{E^{cap}}$, will be varied. FBSDEs of this kind were studied in [1].

Fitting the seasonal factors $\mathbf{h^i}$

We start with the following data:

- Electricity generation due to coal and/or natural gas during 2012-2014,
- 1 month coal futures prices 2012-2014,
- 'Within day' natural gas futures prices 2012-2014.

All time series exhibit strong seasonality, mean reversion and spikes. For example, see Figure 1 for the generation data. The following function $\mathbf{h^D}$, was fitted, using nonlinear regression, to the log generation data.

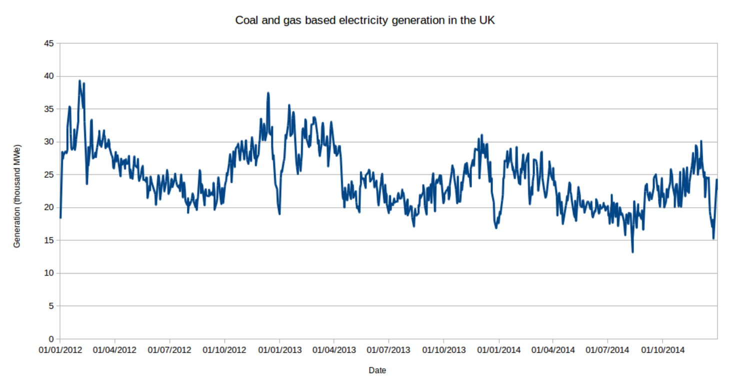

Figure 1: Coal and gas-based electricity generation (2012-2014)

$$\mathbf{h^D}(\mathbf{t}) = \mathbf{a^D} + \mathbf{b^D t} + \mathbf{c^D_1} \cos\left(\frac{\pi \mathbf{t}}{2}\right) + \mathbf{d^D_1} \sin\left(\frac{\pi \mathbf{t}}{2}\right) + \mathbf{c^D_2} \cos(2\pi \mathbf{t}) + \mathbf{d^D_2} \sin(2\pi \mathbf{t}).$$

- A similar procedure was used for the coal prices and gas prices.
- The residuals from the fit of $\mathbf{h^i}$ are 'deseasonalized processes', an observation of $\mathbf{X^i}$. Augmented Dickey-Fuller and Phillips-Perron tests suggest that they are stationary.

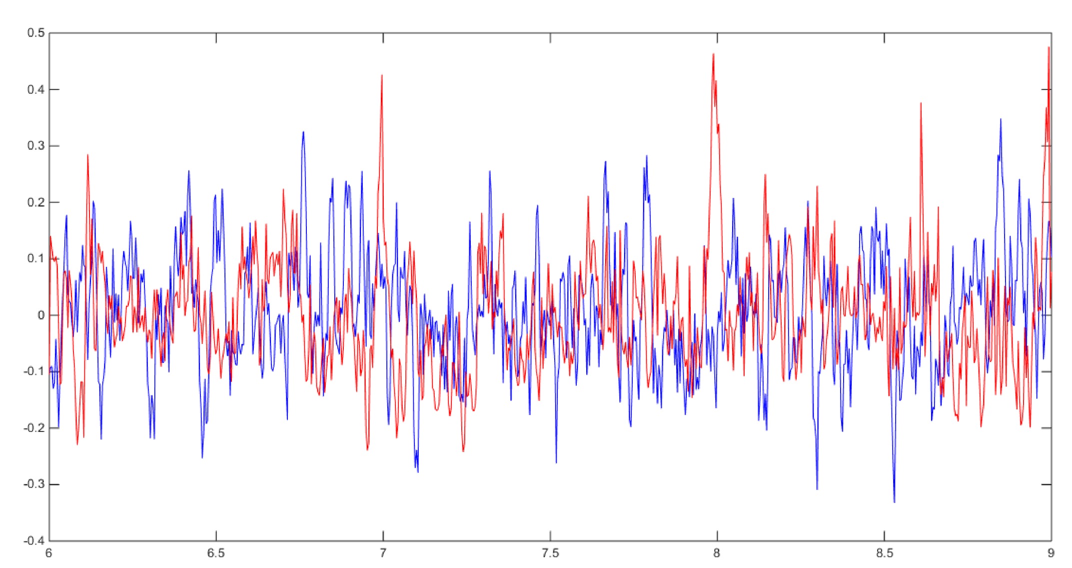

Figure 2: One simulated path (blue) of $\mathbf{X^D}$: an Orstein-Uhlenbeck process with estimated parameters, and observed deseasonalized generation series (red)

Fitting the diffusion processes $\mathbf{X^i}$

Now, we fit appropriate processes $\mathbf{X^i}$ to the deseasonalized time series. For natural gas prices, we choose the process

$$d\mathbf{X^G_t} = -\lambda^G \mathbf{X^G_t} dt + \sqrt{\mathbf{v^G}(\mathbf{X^G_t})} d\mathbf{W^G_t}, \qquad (2)$$

where $\mathbf{v^G}$ was chosen so that the SDE (2) has the density of a Normal-Inverse-Gaussian distribution as its invariant density (see [2]). This is more flexible than the Normal distribution.

- Similar procedure for $\mathbf{X^D}$ and $\mathbf{X^C}$, but with Ornstein-Uhlenbeck processes.
- See Figure 2 for a sample path of one of the resulting diffusion processes.

Specification of μ_E

- We use the bid-stack approach, as in [4]. Emissions regulation changes the marginal cost of producing electricity by a factor of the emissions rate (coal has a higher rate than gas) multiplied by the allowance price.
- Specifying linear functions for the aforementioned, we use UK electricity and EUA (European Union allowance) price data to estimate their parameters.
- Then, μ_E can be obtained by integrating the relevant emissions rates.

Numerical results

- We use the Markovian iteration scheme of [3] for a numerical solution of FBSDE (1).
- Set $\mathbf{\bar{E}^{avg}} = \mathbf{T}\mu_E(\exp(\mathbf{a^D}), \exp(\mathbf{a^C}), \exp(\mathbf{a^G}), \mathbf{0})$. It represents an average level of total emissions when the allowance price is frozen at $\mathbf{0}$.
- Compare the results of the following 4 market setups: no emissions regulation (BAU - business as usual), $\mathbf{E^{cap}} = \mathbf{0.5\bar{E}^{avg}}$, $\mathbf{E^{cap}} = \mathbf{0.6\bar{E}^{avg}}$, $\mathbf{E^{cap}} = \mathbf{0.7\bar{E}^{avg}}$.
- See Table 1 below for the values of $\mathbf{A_0}$, $\mathbb{E}[\mathbf{E_T}]$ and $\mathbb{P}[\mathbf{E_T} \geq \mathbf{E^{cap}}]$, and Figure 3 for some sample paths.

Table 1: Simulation results for 3 different model setups and the business-as-usual setting

	$\mathbf{A_0}$	$\mathbb{E}[\mathbf{E_T}]$ (millions)	$\mathbb{P}[\mathbf{E_T} \geq \mathbf{E^{cap}}]$
(Business-as-usual)	(NA)	261.113	(NA)
$\mathbf{E^{cap}} = \mathbf{0.7\bar{E}^{avg}}$	0.261	261.408	0.003
$\mathbf{E^{cap}} = \mathbf{0.6\bar{E}^{avg}}$	14.005	259.795	0.167
$\mathbf{E^{cap}} = \mathbf{0.5\bar{E}^{avg}}$	59.685	252.20	0.716

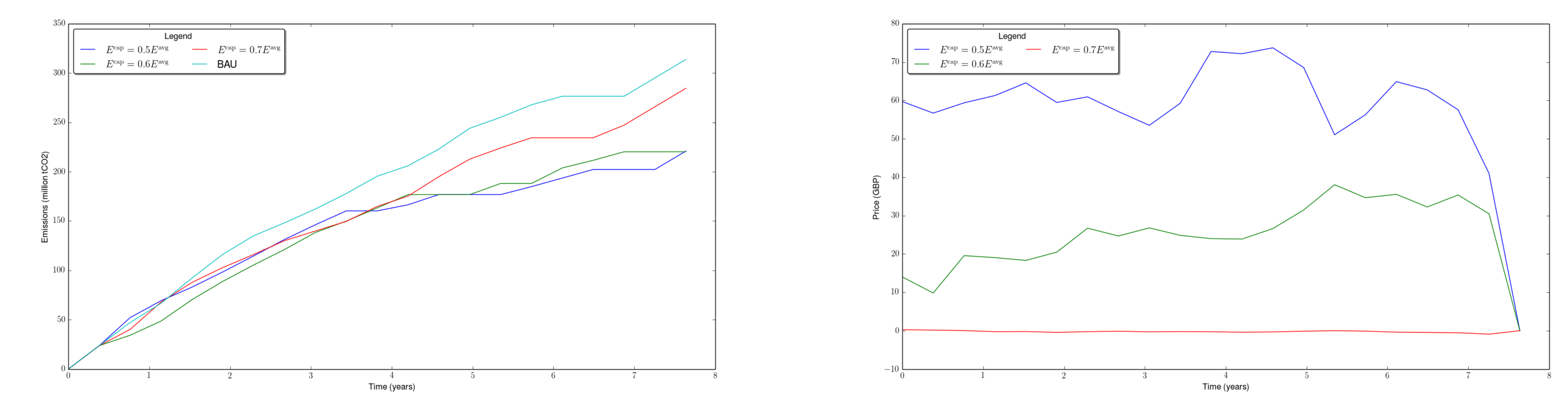

Figure 3: Sample paths of cumulative emissions $\mathbf{E_t}$ (left) and allowance price $\mathbf{A_t}$ (right)

- **Conclusion**: Stricter caps lead to higher allowance prices and greater emissions reduction. A very lenient cap will not cause any emissions reduction.
- **Future research**: Repeat the analysis with a multi-period emissions trading model.

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