Scaling Cascades in Complex Systems

Non-Gaussian data assimilation via a localized hybrid ensemble transform filter

Walter Acevedo, Nawinda Chutsagulprom, Maria Reinhardt and Sebastian Reich
Universität Potsdam/ University of Reading

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Outline

Motivation

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Hybrid Scheme

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Ensemble Transform Particle Filter [Reich and Cotter, 2015]

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- Ensemble Inflation and Particle Rejuvenation

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Conclusions and Prospect

Sequential Data Assimilation Strategies

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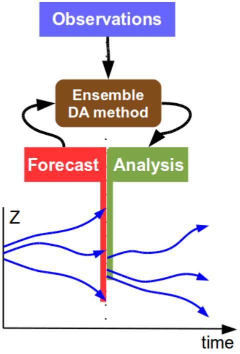

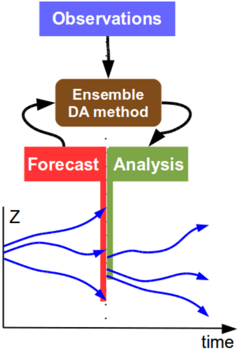

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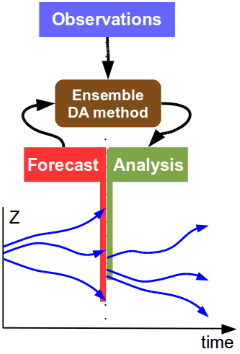

- Ensemble Kalman Filters (Gaussian Approach)
  - + Robust

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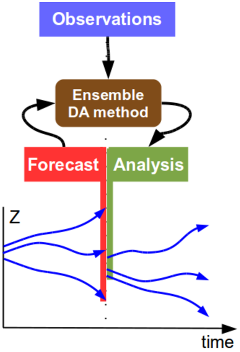

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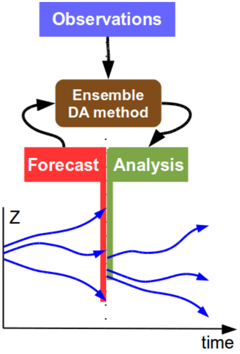

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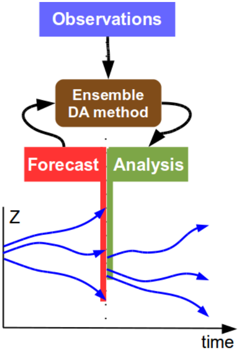

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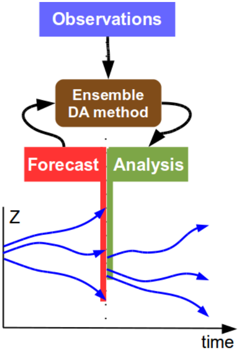

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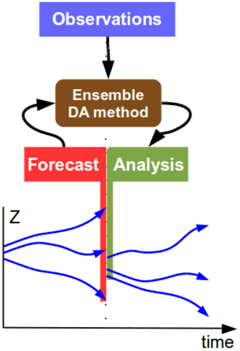

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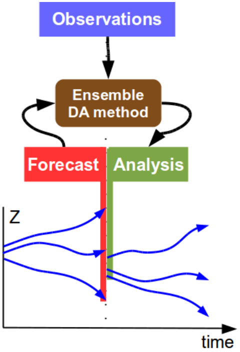

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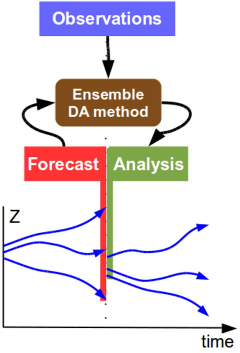

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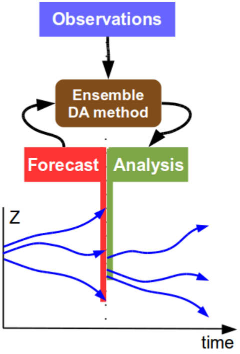

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- Hybrid schemes
  - + trade-off between accuracy and stability

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Ensemble Transform Particle Filter [Reich and Cotter, 2015]

Importance weights: $$w_i = \frac{\exp\left(-\frac{1}{2}(Hz_i^{\mathrm{f}} - y_{\mathrm{obs}})^{\mathrm{T}} R^{-1} (Hz_i^{\mathrm{f}} - y_{\mathrm{obs}})\right)}{\sum_{j=1}^{M} \exp\left(-\frac{1}{2}(Hz_j^{\mathrm{f}} - y_{\mathrm{obs}})^{\mathrm{T}} R^{-1} (Hz_j^{\mathrm{f}} - y_{\mathrm{obs}})\right)} \quad (1)$$

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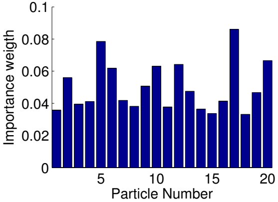

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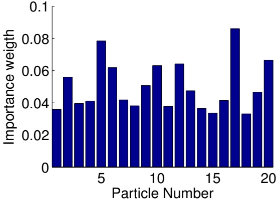
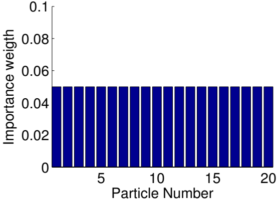


$\Rightarrow$ Optimal Coupling T^* $\Rightarrow$



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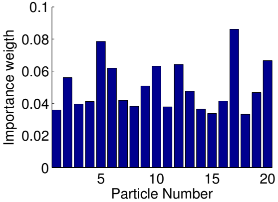

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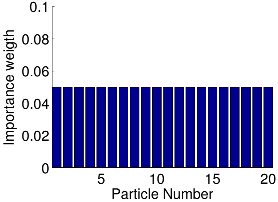

$T^* : \{t_{ij}^* \geq 0\}$ is a linear transformation: $$z_j^{\mathrm{a}} = M \sum_{i=1}^{M} z_i^{\mathrm{f}} t_{ij}^* \quad (2)$$

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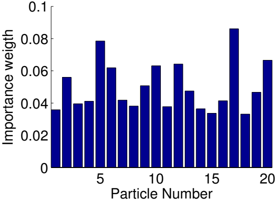

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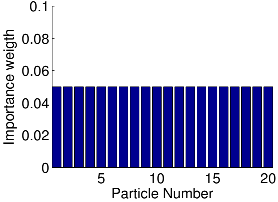

$T^* : \{t_{ij}^* \geq 0\}$ is a linear transformation: $$z_j^{\mathrm{a}} = M \sum_{i=1}^{M} z_i^{\mathrm{f}} t_{ij}^* \quad (2)$$

which minimizes the cost function: $$J(\{t_{ij}\}) = \sum_{i,j=1}^{M} t_{ij} \| z_i^{\mathrm{f}} - z_j^{\mathrm{f}} \|^2 \quad (3)$$

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Likelihood function splitting

$$\pi_Y(y_{\text{obs}}|z) \propto$$

$$\exp\left(-\frac{\alpha}{2}(Hz - y_{\text{obs}})^{\mathrm{T}} R^{-1} (Hz - y_{\text{obs}})\right) \times$$

$$\exp\left(-\frac{1-\alpha}{2}(Hz - y_{\text{obs}})^{\mathrm{T}} R^{-1} (Hz - y_{\text{obs}})\right)$$

with bridging parameter $\alpha \in [0, 1]$

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$$\exp\left(-\frac{\alpha}{2}(Hz - y_{\text{obs}})^{\mathrm{T}} R^{-1}(Hz - y_{\text{obs}})\right) \times \quad \Rightarrow \text{ETPF}$$

$$\exp\left(-\frac{1-\alpha}{2}(Hz - y_{\text{obs}})^{\mathrm{T}} R^{-1}(Hz - y_{\text{obs}})\right) \quad \Rightarrow \text{EnKF, ETKF, ESRF,} \ldots$$

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Related work

Ensemble Kalman Particle Filter (EKPF) [Frei and Künsch, 2013]: bridges the EnKF with perturbed observations and a SIR particle filter.

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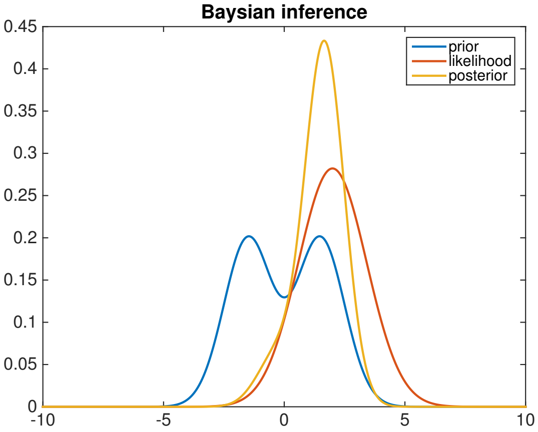
Example: Single 1D Assimilation Step

Bayesian Inference

for bimodal prior

and

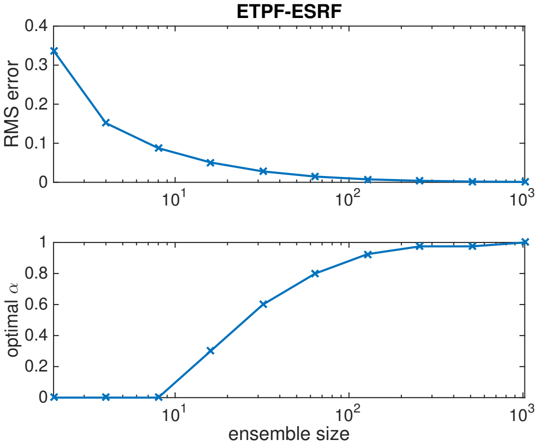
Gaussian likelihood



ETPF-ESRF

performance vs

ensemble size



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Scaling Cascades in Complex Systems

- **Fixed**:
  keeping bridging parameter constant

Likelihood splitting strategies

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  keeping ETPF effective sample size constant

$$M_{\text{eff}}^{ETPF}(\alpha) = \frac{1}{\sum_{i=1}^{M} w_i(\alpha)^2} = \theta M \qquad (4)$$

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Two possible updating orders!

Scaling Cascades in Complex Systems

In order to prevent

- Ensemble under-dispersion

Ensemble Inflation and Particle Rejuvenation

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- β: Rejuvenation parameter
- ξ_{ij}'s: i.i.d. Gaussian random variables with mean zero and variance one
- $\sum_{j=1}^{M} \xi_{ij} = 0$ so as to preserve the ensemble mean

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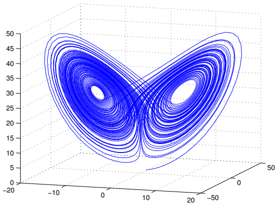
Example: Lorenz 63 model

Scaling Cascades in Complex Systems

$$\dot{x}_1 = 10(x_2 - x_1)$$
$$\dot{x}_2 = x_1(28 - x_3) - x_2$$
$$\dot{x}_3 = x_1 x_2 - 8/3 x_3$$



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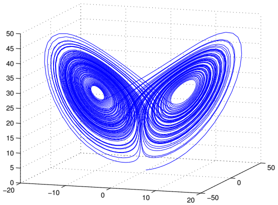

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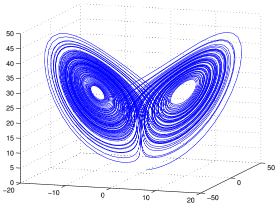

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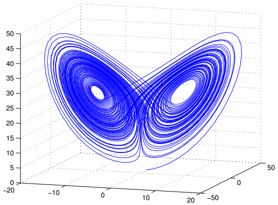

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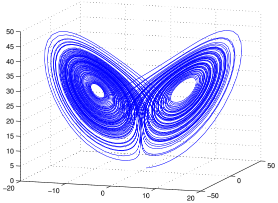

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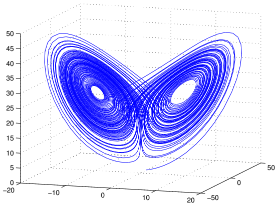
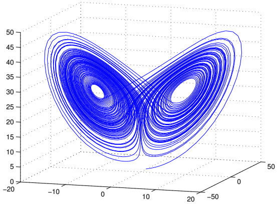


Perfect model DA experiments:

- Implicit midpoint method with time step $\Delta t = 0.01$.
- x_1 observed every 12 time-steps
- Particle rejuvenation $\beta = 0.2$
- 100,000 assimilation cycles

Example: Lorenz 63 model

$$\dot{x}_1 = 10(x_2 - x_1)$$
$$\dot{x}_2 = x_1(28 - x_3) - x_2$$
$$\dot{x}_3 = x_1 x_2 - 8/3 x_3$$

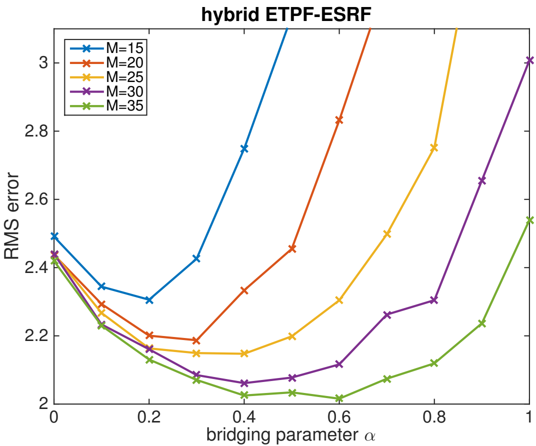


Perfect model DA experiments:

- Implicit midpoint method with time step $\Delta t = 0.01$.
- x_1 observed every 12 time-steps
- Particle rejuvenation $\beta = 0.2$
- 100,000 assimilation cycles
- OTP solved using FastEMD algorithm

Scaling Cascades in Complex Systems

Skill dependence on bridging parameter for different ensemble sizes using fixed likelihood splitting



Example: Lorenz 63 model

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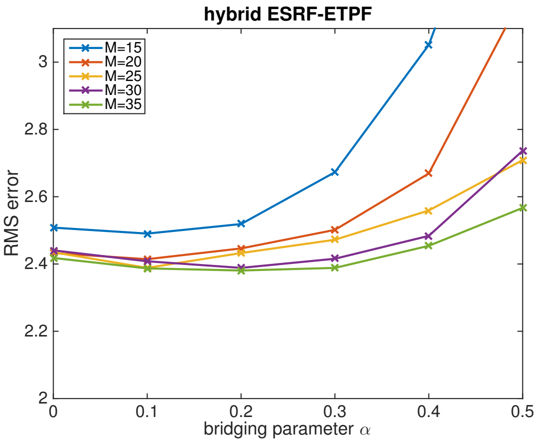
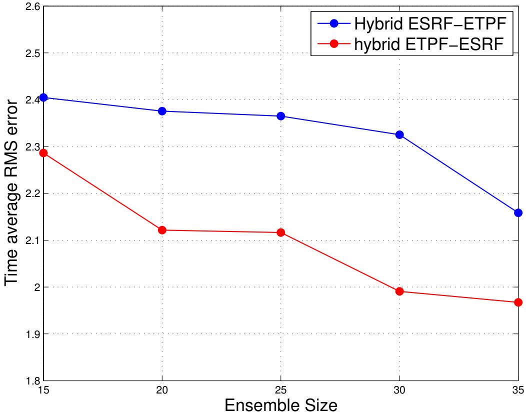
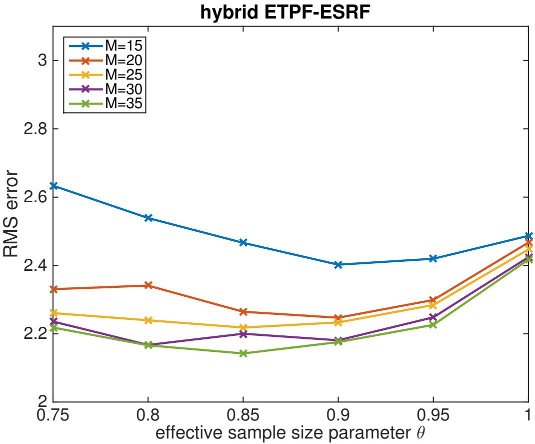


Skill dependence on ensemble size for both update orders using optimal fixed bridging parameter



Skill dependence on effective ETPF sample size for different ensemble sizes using adaptive likelihood splitting



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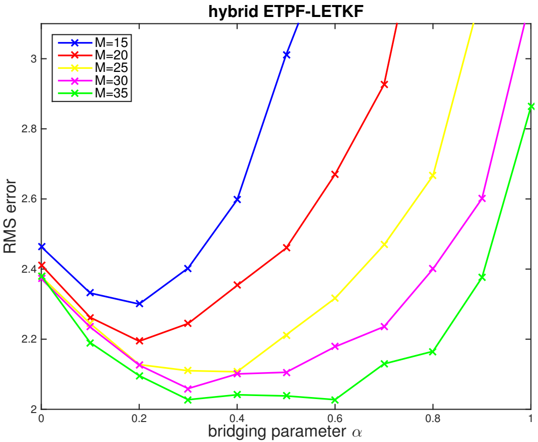

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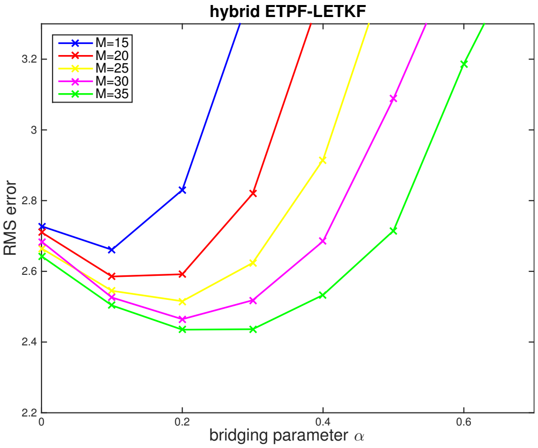

Outline

Scaling Cascades in Complex Systems

Motivation

Hybrid Scheme
- Ensemble Transform Particle Filter [Reich and Cotter, 2015]
- Hybrid Ensemble Transform Filter [Chustagulprom et al., 2015]
- Example: Single 1D Assimilation Step

Non-spatially extended systems
- Likelihood splitting strategies
- Ensemble Inflation and Particle Rejuvenation
- Example: Lorenz 63 model

Spatially extended systems
- **Localization**
- Example: Lorenz 96 model

Conclusions and Prospect

R-Localization

Localized measurement error covariance elements:

$$r_{qq}^{LOC}(x_k) = \frac{r_{qq}}{\rho\left(\frac{\|x_k - x_q\|}{R_{\text{loc}}}\right)},$$

with ρ a compactly supported tempering function, e.g., Gaspari-Cohn function.

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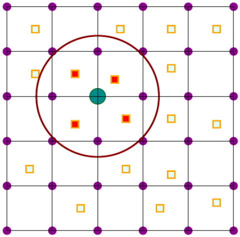

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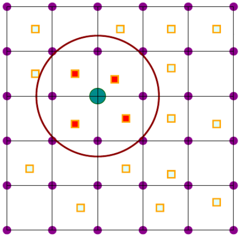

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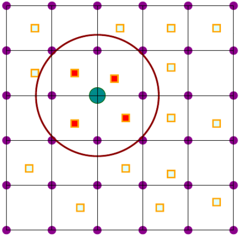

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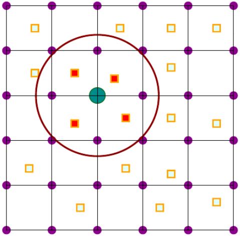

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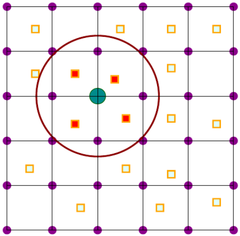

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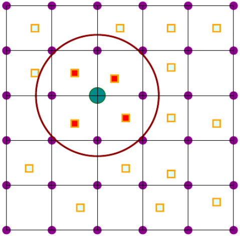

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  Local importance weights and cost function:

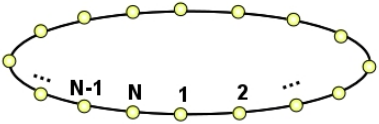
$$J(\{t_{ij}\}, x_k) = \sum_{i,j=1}^{M} t_{ij} \int_{\mathbb{R}} \rho\left(\frac{\|x_k - x_q\|}{R_{\text{loc}}}\right) \|z_i^{\text{f}}(x_q) - z_j^{\text{f}}(x_q)\|^2 dx_q \qquad (6)$$

Outline

Example: Lorenz 96 model

$$\dot{x}_j = (x_{j+1} - x_{j-2})x_{j-1} - x_j + F,$$
$$x_j = x_{j+N}$$

where $F = 8$ and $N = 40$.



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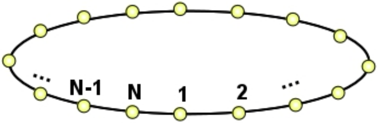

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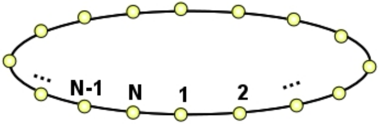

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Scaling Cascades in Complex Systems

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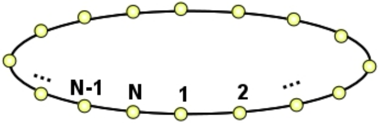

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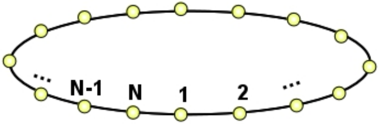

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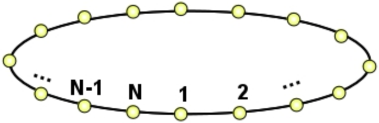

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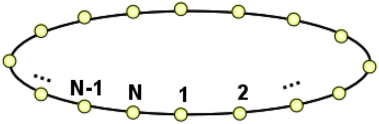

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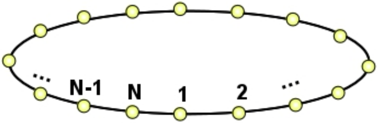

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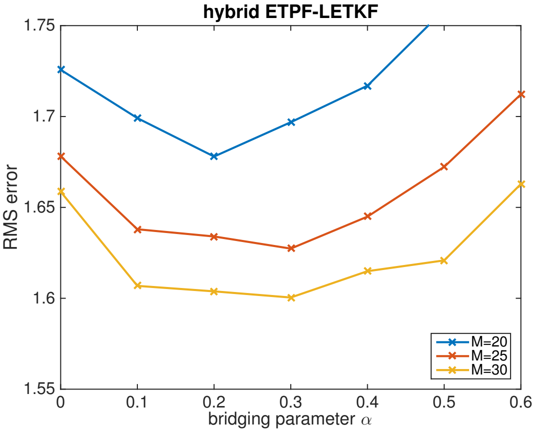

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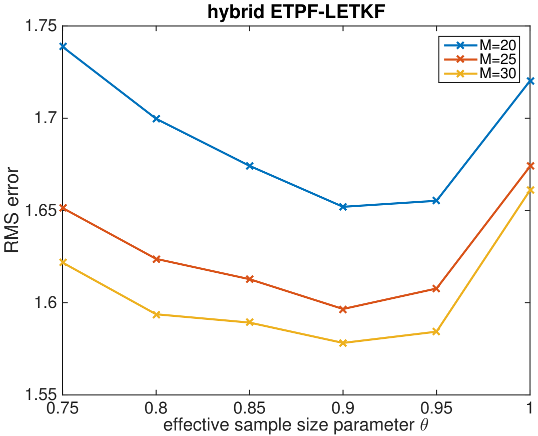
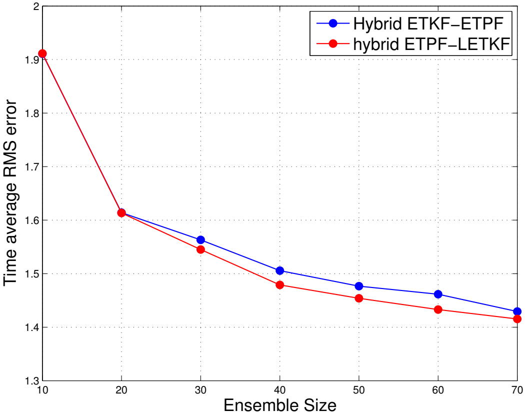
Skill dependence on effective ETPF sample size for different ensemble sizes using adaptive likelihood splitting



Skill dependence on ensemble size for both update orders using optimal fixed bridging parameter



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- Adaptive likelihood splitting was beneficial for Lorenz-96 system and detrimental for Lorenz-63 model. Other adaptation criteria must be tested

Ongoing Research and Prospect

- Hybrid scheme being implemented into the DWDs DA system

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Thanks!